Linear Transformations

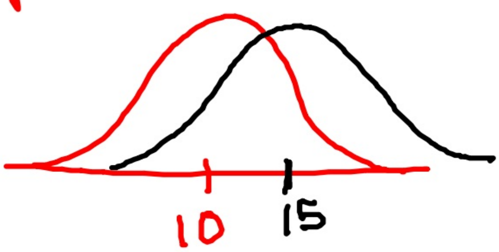
Shape = same

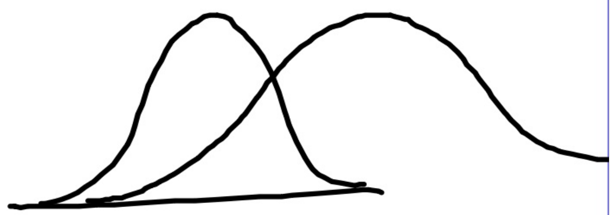
Center: +, −, ×, ÷

Spread: ×, ÷

×a all

+b center only

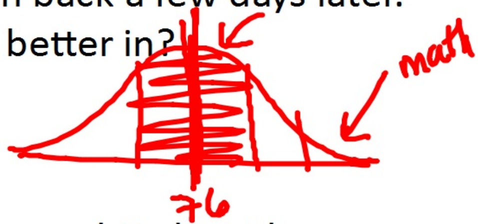




Standardizing Observations

1) You are in a history class and a math class. You take your chapter 1 test in both classes on the same day, and get them both back a few days later. Your grades are as follows. Which class did you do better in?

History: 81% *Math: 75%*



2) Same scenario. However, each teacher tells you a bit about how your class did. Which class did you do better in?

History: 81% *Math: 75%*

mean: 76% *mean: 70%*

either

3) Same scenario. However how your teacher gives you the class standard deviations. Which class did you do better in?

less than 1s

History: 81% *Math: 75%*

mean: 76% *mean: 70%*

std. dev: 8% *std. dev: 2.5%*

2s

Standardizing Observations

Question: How can we compare one distribution to another distribution if they don't have the same parameters (mean and std. dev)?

Answer: compare the observation to its mean & std. dev.

To standardize:

- measure observations.... in terms of how many std. dev.'s they are from mean.
- Z-score = $\dfrac{X - \bar{X}}{S}$
- Z- score tells us... how many S above or below $\bar{X}$.

math = $\dfrac{75-70}{2.5} = \underline{2}$

hist = $\underline{0.625}$

Example: The heights of 18-24 year old women are distributed with the following: μ = mean of a pop.

μ = 64.5" and σ = 2.5" σ = std. dev of pop.

We know a woman who is 69" tall. How does she compare to the rest of the women in her age group? Calculate her z score, and interpret what it means.

$$z = \frac{x - \mu}{\sigma} = \frac{69 - 64.5}{2.5} = 1.8\sigma \text{ above } \mu$$

Example: We also know a man who is 71" tall. Who is taller relatively? Men are known to be distributed with a mean of 67" and a std. deviation of 2.3 inches.

woman = 1.8σ

man = 1.74σ

Changing/Manipulating a set of data

What did we learn from the linear transformations wksht??

Adding/Subtracting a number just changes… centers

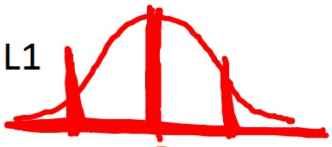
Dividing/Multiplying a number changes…. center + spread

Did the shape change in any of these??? NO!

Example:

$\bar{x}$

Set of data: {2, 3, 3, 4, 4, 4, 5, 5, 6} Type this into L1



0

1) Find the mean and std. deviation and the shape

$\bar{x} = 4$ $s = 1.2247$ shape = sym.

2) Take the entire set of data and convert to z-scores (subtract the mean and then divide by the std. deviation). Do this in L2.

$L2 = (L1 - 4)/1.2247$

3) Calculate the new mean and std. dev (of L2). Why is this so?

$\bar{x} = 0$ $s \approx 1$

4) What does the shape of the distribution look like?

Symm.

This will happen with ANY distribution.... When you change the whole thing into z-scores, your mean will be ___0___ and your std. dev. will be ___1___.

Graphs for Samples:

histograms, stemplots, etc.

Described by STATISTICS

$\bar{x}$, s, M

Model for a population:

$\bar{x} = \$45,000$

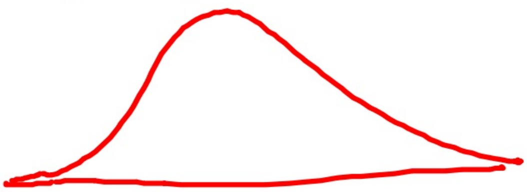
Described by PARAMETERS

μ

σ

~~M~~

Specific type of population.... NORMAL



$N(64.5, 2.5)$

Notation: $N(\underline{\mu}, \underline{\sigma})$

Notes on Standardizing a distribution: whole pop. or sample

- Standardizing one observation…

compares it to it's mean

- Standardizing a whole distribution allows us to….

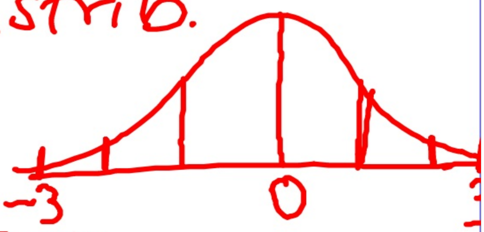
- compare whole distrib. to it's mean

⇒ + compare to another distrib.

- Normal distributions…. When we standardize them, we get the:

Standard Normal Distrib.

$N(0, 1)$



- Can only be used for…

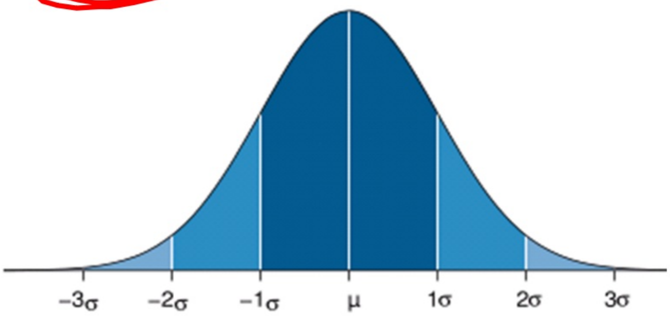
nearly normal data/pop

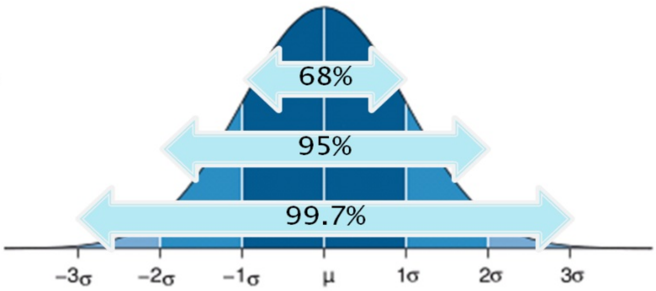
- How can we check if data is NORMAL?

histogram

Empirical Rule

In a normal distribution with N(μ, σ)....





- 68 % of the observations fall within $\mu \pm \sigma$
- 95 % of the observations fall within $\mu \pm 2\sigma$
- 99.7 % of the observations fall within $\mu \pm 3\sigma$

Example 1: WHO GETS THE GOLD??

	Event		
Competitor	**100m Dash**	**Shot Put**	**Long Jump**
A	0.5 10.1 sec	2 66'	0 26'
B	-0.5 9.9 sec	0 60'	0.167 27'
C	1.5 10.3 sec	1 63'	0.2083 27'3"
Mean	10 sec	60'	26'
St Dev	0.2 sec	3'	6"

Example 2:

To be certain that you would be accepted to the college you want, you would need an SAT score of at least 1900. However you took the ACT. What score would you need to get?

For SAT:	mean = 1500	s = 250
For ACT:	mean = 20.8	s = 4.8

Example:

The avg. time that it takes a crew from Sweeper Sam to clean a room is normally distributed with a mean of 2.1 hrs and a standard deviation of 0.3 hrs.

$N(2.1, 0.3)$

a) The total clean up time will fall within what interval 95% of the time?

$\mu \pm 2\sigma = (1.5, 2.7)$ hrs.

b) What proportion of the time will it take the crew 2.~~5~~4 hours or more?

%

$2.4 = \mu + \sigma$

c) What percent of the time will it take the crew 1.5 hrs or less?

Back to the height example….

Remember that the heights of 18-24 year old women are N(64.5", 2.5"). What percentile is the girl who is 68" tall?

% below

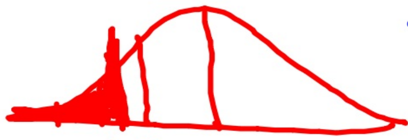
$z = 1.4$

Calculator: normalcdf(lower bound, upper bound, μ, σ)

What percent of 18-24 year old women are less than 5 feet tall?

normalcdf(−E99, −1.8, 0, 1)

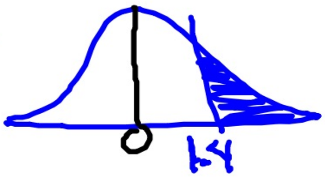
$$z = \frac{60-64.5}{2.5} = -1.8$$



3.59%

What percent 18-24 year old of women are over 5'8" tall?

$$z = \frac{68-64.5}{2.5} = 1.4$$



normalcdf(1.4, E99, 0, 1)

8.076%

**** PROBABILITY NOTATION!!**

$N(\mu, \sigma)$

$P(Z < 1.4) = P(X < 68'') = ?$

Another example:

Blood pressures of high school students are N(170, 30). What is the probability that a randomly selected high school student has a blood pressure of 180 or above?

$P(X \geq 180) =$ $z = 0.333$

36.94%

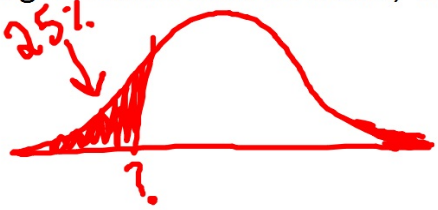
Using the same data as above, what is the probability that a high school student has a blood pressure between 160 and 230?

$\boxed{P(160 < X < 230)}$

$z = -0.333$ $z = 2$

60.78%

Using the same data as above, what blood pressure has 25% of the observations below it?



$-0.674 = \dfrac{X - 170}{30}$

$P(X < ?) = 0.25$

Calculator use:

To find the PERCENT of observations BETWEEN 2 points:

normalcdf(*lower bound, upper bound, mean, std. dev.*)

To find what OBSERVATION has a certain percent of the data BELOW it:

invnorm(*percent below, mean, std. dev.*)

On the calculator, INFINITY is:

E99 = infinity
-E99 = -infinity

Complete worksheet 6A and check answers on the front desk!